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Twist-three distributions and their appearance in the doubly-polarized Drell-Yan process[1]

R.D. Tangerman and P.J. Mulders

National Institute for Nuclear Physics and High Energy Physics (NIKHEF), P.O. Box 41882, 1009 DB Amsterdam, The Netherlands

Abstract

The twist-three distributions $g_2(x)$ and $h_2(x)$ are defined as quark-field matrix elements between polarized hadron states. They can be written in terms of quark-mass and gluonic operators, after which the Burkhardt-Cottingham sum rule for g_2 can be derived and a similar sum rule for h_2. Their role in the Drell-Yan double-spin asymmetry A_{LT} is explained.

[1]Talk presented by R.D. Tangerman at 2nd Meeting on Possible Measurements of Singly Polarized pp and pn Collisions at HERA (DESY-IfH Zeuthen, August 31 - September 2, 1995).

1 Introduction

Although the basic ingredients for high-energy scattering experiments, the quark distribution functions defined as quark-field matrix elements, can not be calculated from QCD directly, certain sum rules have a more general origin. The Adler sum rule, for instance, follows from current conservation. At sub-leading twist, one has the Burkhardt-Cottingham (BC) sum rule for g_2, and a similar sum rule for h_2, which follow from rotational invariance [1, 2]. First attempts have been made to measure the BC sum rule [3]. The measurement of the h_2 sum rule may be possible in the doubly-polarized Drell-Yan process.

Within classes of equal twist, parton distribution functions can be distinguished by their spin-content. For unpolarized hadrons, for instance, one has the light-cone correlator[2]

$$\int \frac{d\lambda}{4\pi}\, e^{i\lambda x}\, \langle P|\overline{\psi}(0)\gamma^\mu\psi(\lambda n)|P\rangle = f_1(x)\, p^\mu + f_4(x)\, M^2 n^\mu, \tag{1}$$

where the hadron momentum reads $P^\mu = p^\mu + (M^2/2)n^\mu$ in terms of the vectors p and n, satisfying $p^2 = n^2 = 0$ and $p\cdot n = 1$. Not an unimportant detail is the use of the light-cone gauge $A\cdot n = 0$, such that the path-ordered link operator $\mathcal{P}\exp[-ig\int_0^\lambda d\mu\, A(\mu n)\cdot n]$, inserted between the quark fields to ensure color gauge invariance, becomes unity. The twist-two momentum distribution $f_1(x)$ will contribute to the leading DIS structure function $F_T(x_B, Q^2)$, the logarithmic Q^2 dependence coming from radiative corrections (GLAP evolution).

If the hadron is polarized with spin vector $S^\mu = (S{\cdot}n)p^\mu + S_T^\mu - (M^2/2)(S{\cdot}n)n^\mu$, the axial vector matrix element is parametrized like

$$\int \frac{d\lambda}{4\pi}\, e^{i\lambda x}\, \langle PS|\overline{\psi}(0)\gamma^\mu\gamma_5\psi(\lambda n)|PS\rangle = g_1(x)\,(S{\cdot}n)p^\mu + g_T(x)\, S_T^\mu + g_3(x)\, M^2(S{\cdot}n)n^\mu, \tag{2}$$

where $g_1(x)$ is the twist-two helicity distribution. In this note we will concentrate on the combination $g_2(x) = g_T(x) - g_1(x)$, containing the transverse spin distribution, and its longitudinal counterpart $h_2(x)/2 = h_L(x) - h_1(x)$ defined by the light-cone correlator

$$\int \frac{d\lambda}{4\pi}\, e^{i\lambda x}\, \langle PS|\overline{\psi}(0) i\sigma^{\mu\nu}\gamma_5\psi(\lambda n)|PS\rangle =$$
$$\frac{h_1(x)}{M}\,(S_T^\mu p^\nu - S_T^\nu p^\mu) + h_L(x)\, M(S\cdot n)(p^\mu n^\nu - p^\nu n^\mu) + h_3(x)\, M(S_T^\mu n^\nu - S_T^\nu n^\mu), \tag{3}$$

where $h_1(x)$ is the twist-two transversity distribution.

[2] We use the nomenclature and conventions of Jaffe and Ji in Ref. [4].

2 Sum rules

The parton interpretation of the higher-twist distribution function is much more involved than for the twist-two case. This arises because of the mixing in the operator-product-expansion of gluonic and quark-mass operators [5, 6]. Consider first the transverse-spin distribution $g_2(x)$. In Ref. [2] it is shown to be closely related to the existence of intrinsic quark transverse momentum in the nucleon. Using the Dirac equation and Lorentz covariance, it can be rewritten as

$$g_2(x) = -g_1(x) + \int_x^1 \frac{dy}{y} g_1(y) + \frac{m}{M}\left[\frac{h_1(x)}{x} - \int_x^1 \frac{dy}{y}\frac{h_1(y)}{y}\right] + \tilde{g}_T(x) - \int_x^1 \frac{dy}{y}\tilde{g}_T(y), \tag{4}$$

where m is the current-quark mass. The distribution $\tilde{g}_T$ contains explicit gluon fields, and is defined by

$$x\tilde{g}_T(x)\, S_T^\mu = \int \frac{d\lambda}{8\pi}\, e^{i\lambda x}\, \langle PS|\overline{\psi}(0) g A_{T\nu}(0)[g_T^{\mu\nu}\, \not{n}\gamma_5 + i\epsilon_T^{\mu\nu}\, \not{n}]\psi(\lambda n)|P\rangle + \text{h.c.}, \tag{5}$$

using the transverse tensors $g_T^{\mu\nu} = g^{\mu\nu} - p^\mu n^\nu - p^\nu n^\mu$ and $\epsilon_T^{\mu\nu} = \epsilon^{\mu\nu\rho\sigma} p_\rho n_\sigma$. The decomposition (4) is interesting, since it gives rise to the Burkhardt-Cottingham sum rule [7]

$$\int_0^1 dx\, g_2(x) = 0, \tag{6}$$

provided the order of x and y integrations may be interchanged. This sum rule has received some attention recently. Specifically, it was shown that for a free-quark target it acquires no $\mathcal{O}(\alpha)$ corrections [8].

In a similar fashion, the longitudinal-spin distribution $h_2(x)$ can be decomposed as

$$\begin{aligned} \frac{h_2(x)}{2} &= -h_1(x) + 2x\int_x^1 \frac{dy}{y^2} h_1(y) + \frac{m}{M}\left[\frac{g_1(x)}{x} - 2x\int_x^1 \frac{dy}{y^2}\frac{g_1(y)}{y}\right] \\ &\quad + \tilde{h}_L(x) - 2x\int_x^1 \frac{dy}{y^2}\tilde{h}_L(y), \end{aligned} \tag{7}$$

with the interaction-dependent function

$$x\tilde{h}_L(x)\, M(S\cdot n) = \int \frac{d\lambda}{8\pi}\, e^{i\lambda x}\langle PS|\overline{\psi}(0) g \not{A}_T(0)\, \gamma_5 \not{n}\psi(\lambda n)|PS\rangle + \text{h.c.}. \tag{8}$$

Equation (7) leads to the sum rule [1, 2]

$$\int_0^1 dx\, h_2(x) = 0, \tag{9}$$

for the *distribution* function h_2. The validity of this sum rule for the *structure* function h_2 was questioned recently by Burkardt [1, 9].

3 Experiments

Writing down sum rules for parton distribution functions is one thing, extracting them from experiment is another. The distribution $g_2(x)$ appears cleanest in the cross section for the scattering of a longitudinally polarized lepton on a transversely polarized hadron, measuring only the scattered lepton momentum. From the handbag diagram in Fig. 1 and two diagrams involving a quark-gluon-quark correlator, one finds that the structure function $g_2(x_B, Q^2)$ at tree-level equals $(1/2)\sum_a e_a^2\, g_2^a(x_B)$, the a denoting quark and antiquark flavors. Since $h_2(x)$ is chirally odd, one needs a *second* hadron to flip helicities. Thus we are forced to consider, for instance, Drell-Yan scattering. The analogue to the handbag diagram is depicted in Fig. 2. After averaging over the transverse momentum q_T of the produced massive photon, the distributions appear pair-wise in the double-spin asymmetry [4, 2]

$$
\begin{aligned}
A_{LT} = \quad & \frac{\sigma(\lambda_A, S_{BT}) - \sigma(\lambda_A, -S_{BT})}{\sigma(\lambda_A, S_{BT}) + \sigma(\lambda_A, -S_{BT})} = \frac{M\lambda_A}{Q}\frac{\sin 2\theta\cos(\phi-\phi_B)}{1+\cos^2\theta} \\
& \times \frac{\sum_a e_a^2 \left\{ g_1^a(x) y \left[g_T^{\bar{a}}(y) + \tilde{g}_T^{\bar{a}}(y)\right] + x\left[h_L^a(x) + \tilde{h}_L^a(x)\right] h_1^{\bar{a}}(y)\right\}}{\sum_a e_a^2 f_1^a(x) f_1^{\bar{a}}(y)}.
\end{aligned} \tag{10}
$$

The helicity of the longitudinally polarized hadron is given by $\lambda_A = S_A \cdot n$. The angles θ and ϕ are of those the lepton axis in the photon rest-frame. Also, $x = P_B \cdot q / P_A \cdot P_B$ and $y = P_A \cdot q / P_A \cdot P_B$. It is clear that $h_2(x)$ cannot easily be extracted from the above expression, since it pre-supposes knowledge of the other distributions. Also, the combinations $g_T + \tilde{g}_T$ and $h_L + \tilde{h}_L$ appear, making life even more complicated. Simply neglecting quark transverse momentum such that (for zero quark mass) $\tilde{g}_T = g_T$ and $\tilde{h}_L = h_L$, as is done in Ref. [4], is an inconsistent approximation [2].

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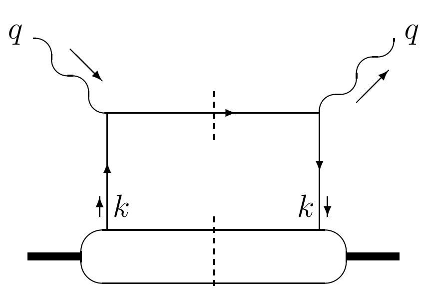

Fig. 1. Handbag diagram.

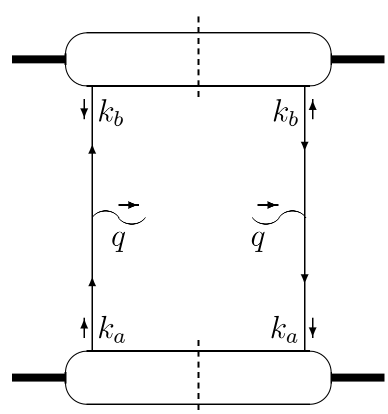

Fig. 2. Drell-Yan tree-level diagram.